**Question 10:**

Let **P(n) ⬄ plus Zero n = n** for any n ∈ Nat

**Base case:**

It is true that **plus Zero Zero = Zero (equivalent to 0 + 0 = 0)** because **plus m Zero = m**

Which is our base case P(0).

**Induction Hypothesis:** Assume P(n)

**Induction**

If **n** is in **P(n)** then **Succ(n)** will be **n+1** in **P(n+1).**

Notice that **plus Zero (Succ(n)) = Succ(plus Zero n)**

whereas **plus Zero n** is our assumption.

Thus ***P*(*n*) *⇒ P*(*n* + 1)**. Alternatively: Thus ***P*(*n* + 1)**

**Conclusion:** By the PMI ∀n ∈ Nat; P(n)

**Question 11:**

Let P(n) <=> plus m n = plus n m

For n, m :: Nat and presume plus n Succ(m) = plus Succ(n) m

**Base case:**

Let P(0) <=> plus m Zero = plus Zero m (substitute n with Zero).

LHS:

plus m Zero

= m line 3

= plus Zero m proved in Question 10.

Thus, P(0).

**Inductive hypothesis**: assume plus n m = plus m n, consider P(n +1) <=> plus m (Succ n) = plus (Succ n) m. We have:

LHS:

plus m (Succ n)

= Succ (plus m n) line 4

= Succ (plus n m) Inductive hypothesis

= plus n (Succ m) line 4

= plus (Succ n) m presumed

Therefore, P(n +1). Thus P(n) follows from the principle mathematical of induction (HI).